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POSSIBILITIES OF AN EXPLOSIVE MHD GENERATOR AS AN ENERGY SOURCE FOR A PLASMA FOCUS

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A plasma focus is presently one of the most powerful sources of neutron and x-ray radiation.

The analysis of experimental results shows that the neutron yield is $N \sim E_0^{\alpha} \sim I^{\beta}$, where E_0 and I are the discharge energy and current [1, 2], $1.5 \leqslant \alpha \leqslant 2.5$, and $\beta \sim 3.3$. Experiments have been conducted mainly with capacitor batteries having energies of up to 400 kJ. The use of explosive MHD generators, which make it possible to obtain a current and energy in the load exceeding their initial values by 10-50 times [3], is promising as the initial energy sources in the megajoule range.

Research on the electrotechnical matching of a specific explosive-magnetic generator with a plasma focus (EG-PF) was reported on in [4]. However, the explosive-magnetic generator under consideration, with a low inductance (400 cm) and a long working time (150 μsec) transferred a small fraction of its energy to the high-temperature plasma of the focus. In addition, the size of the chamber proved to be large: an outside diameter of 2.4 m and an insulator diameter of 90 cm. In the present report computer calculations are made modeling the operation of an explosive MHD generator loaded onto a noncylindrical Z pinch of N. V. Filippov's geometry [5]. The purpose of the work was to investigate the possibility of obtaining values of the current and energy in the plasma of the focus considerably exceeding the initial values of the current and energy in the generator and to determine the parameters of the EG-PF required for this. The motion of the current shell was described by the snowplow model [6]. As noted in [7, 8], the dynamics of the motion of a current shell far from the axis calculated from the snowplow model agrees with the experimental data and the results of two-dimensional MHD calculations. The relatively short duration of the last stage of cumulation of a shell, when the snowplow model is incorrect, should not significantly affect the matching of the times of EG and convergence of the shell. This is confirmed by the agreement between the experimental data and calculations from the snowplow model of a noncylindrical Z pinch supplied from a capacitor battery [9]. In accordance with experimental data on the neutron yield $N \sim E_0^{\alpha} \sim I^{\beta}$ we used two criteria for matching an explosive MHD generator with a plasma focus: obtaining the maximum density of kinetic energy E_k of the shell in the cumulation zone; obtaining the maximum current at the momemt of cumulation.

A diagram of an EG-PF system is given in Fig. 1. Because of axial symmetry the motion of the current shell is analyzed in the (r, z) plane. In Fig. 1, L_1 is the inductance of the generator, L_2 = const is the stray inductance of the circuit, R = const is the total resistance of the circuit, r_0 is the radius of the anode, z_1 is the height of the insulator, ρ is the gas density in the chamber, and **v** is the velocity of the shell. The operation of the system is described by the equations

$$\frac{\partial \mu}{\partial t} = 2\pi\rho r\left(v\frac{\partial r}{\partial \lambda} - u\frac{\partial z}{\partial \lambda}\right),\quad \frac{\partial(\mu u)}{\partial t} = -\frac{1}{c^2}\frac{I^2}{r}\frac{\partial z}{\partial \lambda},\quad \frac{\partial(\mu v)}{\partial t} = \frac{1}{c^2}\frac{I^2}{r}\frac{\partial r}{\partial \lambda},$$
$$\frac{\partial r}{\partial t} = u,\quad \frac{\partial z}{\partial t} = v,\quad \frac{d}{dt}\left(\frac{L}{c^2}I\right) + RI = 0,\quad L = L_1 + L_2 + L_3, \tag{1}$$

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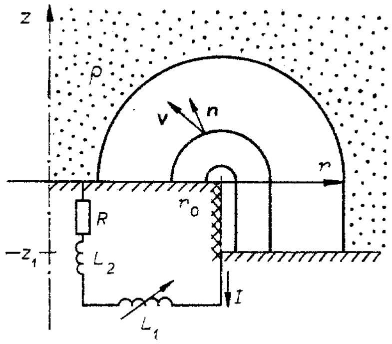

Fig. 1

$$L_1(t) = \begin{cases} L_1^0(1 - t/\tau_0), & 0 \leqslant t \leqslant \tau_0, \\ 0, & \tau_0 \leqslant t, \end{cases}$$

$$L_2 = \text{const}, \quad L_3(t) = -2 \int\limits_{\lambda=0}^{\lambda_3} \frac{\partial z(\lambda, t)}{\partial \lambda} \ln\left(\frac{r(\lambda, t)}{r_0}\right) d\lambda.$$

Here $L_3(t)$ is the inductance of the current shell; $u(\lambda, t)$ and $v(\lambda, t)$ are projections of the velocity vector $\mathbf{v} = (u, v)$; λ is the Lagrangian dimensionless coordinate for points of the shell ($\lambda = 0$ for the left end and $\lambda = \lambda_3 = \text{const}$ for the right end). The mass of the entire shell can be calculated as

$$M(t) = \int\limits_{\lambda=0}^{\lambda_3} \mu(\lambda, t)\, d\lambda,$$

where $\mu(\lambda, t)$ is the mass of the shell per unit of the Lagrangian coordinate. It is assumed that the current shell is formed through the discharge of a current I_0, after which the compression of the magnetic flux in the circuit begins at $t = 0$. The shape of the shell at $t = 0$ is determined by the geometry of the discharge in the N. V. Filippov chamber [9]: a semicircle of radius $r_1 = 0.02\, r_0$ joined to a straight line (see Fig. 1). In the case of a small displacement ($r_1/r_0 \ll 1$) the mass and velocity along the shell are determined by an analytical solution of the snowplow equations with $I = I_0 = \text{const}$. With the accuracy of the quantity $O(r_1/r_0)$ one can obtain $(0 \leqslant \lambda \leqslant \lambda_3)$

$$|\mathbf{v}(\lambda, t)| = v_0 = \frac{I_0}{c r_0 \sqrt{2\pi\rho}}, \quad I_0 = I(t = 0);$$

the vector $\mathbf{v}(\lambda, 0)$ is directed along the normal to the shell. On the semicircle $(0 \leqslant \lambda \leqslant \lambda_1)$ $\mu(\lambda, 0) = \frac{\pi^2 r_1^2 r_0 \rho}{\lambda_1}$, on the straight line $(\lambda_1 \leqslant \lambda \leqslant \lambda_3)$ $\mu(\lambda, 0) = \frac{\pi r_1 z_1 (2r_0 + r_1)\rho}{\lambda_2}$, where λ_1 and λ_2 are the lengths of the semicircle and the straight line in Lagrangian coordinates. To write (1) in dimensionless variables it is convenient to introduce the following dimensionless parameters:

$$p_1 = \frac{v_0 \tau_0}{r_0}, \quad p_2 = r_0/z_1, \quad p_3 = R\tau_0 c^2/L_1^0, \quad p_4 = 2r_0/L_1^0, \quad p_5 = (L_1^0 + L_2)/L_2.$$

In the snowplow model the radius of the focus cannot be determined. However, numerical calculations, as in [9], showed that the optimum of the principal parameter p_1 depends weakly on the radius of the focus and therefore the exact value of the latter is unimportant. In the calculations we used a value of $r(\lambda = 0, t)/r_0 = 0.01$ for the radius of the focus. The problem of the motion of the current shell in the unconfined gas up to the moment of focus was solved numerically. The calculations were made in a wide range of the parameters determining the operation of the EG-PF system:

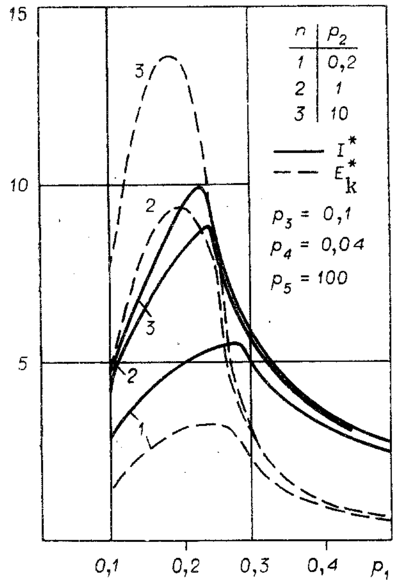

Fig. 2

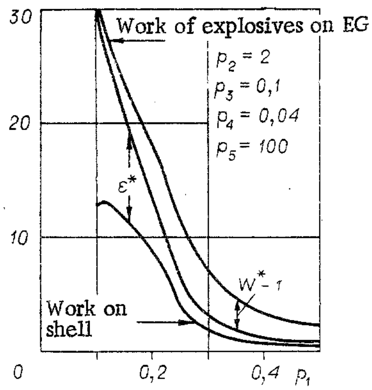

Fig. 3

$$0.1 \leqslant p_1 \leqslant 0.5,\ \ 0.1 \leqslant p_2 \leqslant 10,\ \ 0 \leqslant p_3 \leqslant 0.2,\ \ 0.04 \leqslant p_4 \leqslant 0.5,\ \ 10 \leqslant p_5 \leqslant 100. \tag{2}$$

After trying a number of ways of assigning the boundary conditions for $r(\lambda,\ t = \text{const})$ and $z(\lambda,\ t = \text{const})$ we chose a variant which follows from the continuity of these functions λ together with their derivatives up to the third at the ends of the current shell ($\lambda = 0$, $\lambda = \lambda_3$). Such a variant assured the smoothest shape for the shell at the moment of the pinch and the smallest error in the calculations. We used the method of fractional steps to calculate the time dependence of the quantities μ, r, z, u, v, L_1, L_3, and I. As a control on the calculations we used the law of conservation of energy,

$$\frac{L(0)\,I_0^2}{2c^2} + \begin{pmatrix}\text{work of explosives on}\\ \text{compression of magnet-}\\ \text{ic flux in the explo-}\\ \text{sive generator}\end{pmatrix} = \frac{L(t)\,I^2(t)}{2c^2} + \int_0^t RI^2(t)\,dt + \begin{pmatrix}\text{work of magnetic}\\ \text{field pressure forces}\\ \text{on the current shell}\end{pmatrix},$$

and also checked the equality of the values of the mass of swept-up gas calculated by two methods:

$$M_1 = \int_{\lambda=0}^{\lambda_3} \mu\, d\lambda \text{ and } M_2 = \rho \cdot (\text{swept-up volume})$$

In all the calculations the discrepancy in the law of conservation of energy was less than 1% of $W_0 = L_1^0 I_0^2/2c^2$, while $M_1 \simeq M_2$ with an accuracy no worse than 0.2%.

The calculations showed that in the entire range of the parameters the outside radius of the chamber can be determined in the form

$$r_2 \simeq 1.5 r_0,$$

while the distance from the anode to the cover of the chamber is $z_2 \simeq 0.6 r_0$.

In Fig. 2 we present the kinetic energy density $E_k^* = E_k(\lambda = 0) r_0/W_0$ in the focus zone and the current $I^* = I/I_0$ at the moment of cumulation as functions of the parameter p_1. The two criteria give about the same range of the parameter $p_1 = 0.2 \pm 0.1$ in which the most efficient transfer of the energy of the current shell occurs.

The amounts of the different kinds of energy at the moment of focus as functions of p_1 are shown in Fig. 3. Here

TABLE 1

	Variant			
	I	II	III	IV
I_0, MA	0,25	0,5	0,25	0,5
P, mm Hg	2,5	10	5	20
r_0, cm	30		25	
z_1, cm	3		2,5	
r_1, cm	44		37	
z_2, cm	18		15	
W_0, MJ	0,031	0,125	0,031	0,125
Quantities at moment of cumulation				
Work of explosives on generator, MJ	$32W_0$		$35W_0$	
	1	4	1,1	4,4
Work of magnetic field pressure forces on current shell, MJ	$23W_0$		$24W_0$	
	0,72	2,9	0,75	3
Magnetic energy in circuit, MJ	$8{,}6W_0$		$10W_0$	
	0,27	1,1	0,31	1,3
Magnetic energy in current shell, MJ	$7{,}8W_0$		$9{,}1W_0$	
	0,25	1	0,28	1,1
Joule energy dissipated in resistive component of circuit, MJ	W_0		$1{,}1W_0$	
	0,03	0,125	0,034	0,14
Current at moment of focus, MA	$8{,}9I_0$		$10I_0$	
	2,2	4,4	2,5	5
Maximum current (at moment of end of generator), MA	$27I_0$		$30I_0$	
	6,8	13,5	7,5	15

$$\varepsilon^* = \left(\int_0^{t_1} RI^2dt\right)\Big/W_0, \quad W^* = \left(\frac{L(t_1)I^2(t_1)}{2c^2}\right)\Big/W_0,$$

and t_1 is the moment of formation of the plasma focus.

The calculations determined the optimum range of values of $p_1 \approx 0.2 \pm 0.1$ and showed that the optimum p_1 depend weakly on the variation of p_2, p_3, p_4, and p_5 in the investigated region (2) of their values. This allows us to obtain an equation for matching the EG-PF:

$$p_1 = \frac{v_0\tau_0}{r_0} = \frac{I_0\tau_0}{cr_0^2\sqrt{2\pi\rho}}, \text{ from which } r_0 = \left(\frac{I_0\tau_0}{p_1 c\sqrt{2\pi\rho}}\right)^{0.5}. \quad (3)$$

For a chamber filled with D_2, taking $p_1 = 0.15$, we obtain

$$r_0 = 23(I_0\tau_0 P^{-0.5})^{0.5},$$

where $[I_0] = A$; $[\tau_0] = sec$; $[P] = mm\ Hg$; $P = P_0/\rho_0 \cdot \rho$; P_0 and ρ_0 are the pressure and density of D_2 under standard conditions. A number of variants of a matched EG-PF system are presented in Table 1. For all the variants $L_1^0 = 10^{-6}$ H, $\tau_0 = 10^{-5}$ sec, $L_2 = 10^{-8}$ H, and $R = 10^{-3}\ \Omega$. For $0.3 > p_1 > 0.15$ it is easier to achieve the matched mode (3) of EG-PF operation experimentally.

We note the small radius of the anode in comparison with a chamber supplied by a capacitor battery with similar energetics, as well as the excess of the final values of the current and energy in the shell at the moment of focus over the initial values by 10 and 24 times, respectively.

Thus, the calculations show that considerable energy from an explosive MHD generator can be transferred to a PF plasma.

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